(7 pages) **Reg. No. :** ....

Code No. : 30341 E **Sub. Code : JMMA 62/ JMMC 62**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Sixth Semester

Mathematics/Mathematics with CA – Main

COMPLEX ANALYSIS

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. At $z = 0$, the function $f(z) = |z|^2$ is ————.

   (a) analytic

   (b) differentiable

   (c) not differentiable

   (d) not continuous

2. Complete form of C.R equations is

(a) $f_x = ify$ (b) $f_x = i\dfrac{\partial f}{\partial x}$

(c) $f_x = -if_y$ (d) $f_x = i\dfrac{\partial^2 f}{\partial x^2}$

3. If C is a circle with centre 'a' and radius 'r', then the value of $\int_C \dfrac{dz}{z-a} =$ ____________.

(a) $2\pi i$ (b) $-2\pi i$

(c) 0 (d) $2\pi r$

4. If C is a circle $|z| = r$, then the value of $\int_C \dfrac{dz}{z}$ is ____________.

(a) πi (b) $2\pi i$

(c) 2π (d) π

5. $\lim\limits_{z=0} \dfrac{\sin z}{z} =$ ____________.

(a) 0 (b) 1

(c) ∞ (d) -1

6. The singularities of $\frac{1}{z(z-i)}$ are ______________.

(a) 0 and 1 (b) 0 and i

(c) 1 and 2 (d) 2 and 0

7. To evaluate $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$, which we substitute for z is ______________.

(a) $z = e^{-i\theta}$ (b) $z = e^{i\theta}$

(c) $z = 2e^{i\theta}$ (d) $z = 2e^{-i\theta}$

8. Let $f(z) = \frac{z^2 + 1}{(z^2 + 2z + 2)^2}$. Then ______________ and ______________ are zeros of order 1.

(a) i and $-i$

(b) $1 - i$ and $1 + i$

(c) $-1 + i$ and $-1 - i$

(d) none of these

9. The fixed points of $w = z + b$ is $Z =$ ______________.

(a) 0 (b) ∞

(c) 0 and ∞ (d) 1

10. $w = iz$ represents a rotation through an angle ____________.

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 2π

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 pages.

11. (a) Verify C.R equations for the function $f(z) = e^{-x}(\cos y - i \sin y)$.

Or

(b) Show that if u and v are conjugate harmonic functions, uv is a harmonic function.

12. (a) Prove that $\left| \int_a^b f(t)\, dt \right| \leq \int_a^b |f(t)|\, dt$.

Or

(b) Prove that $\int_{-C} f(z)\, dz = -\int_C f(z)\, dz$.

13. (a) Obtain the Taylor's series to represent $\frac{1}{(z+1)(z+3)}$ in $|z|<1$.

Or

(b) Use Laurent's series, find residue of $\frac{e^{2z}}{(z-1)^2}$ at $z=1$.

14. (a) Using Contour integration, find the value of $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

15. (a) Find the image of the circle $|z-3i|=3$ under the map $w=\frac{1}{2}$.

Or

(b) Show that the transformation $w=\frac{5-4z}{4z-2}$ maps the unit circle $|z|=1$ into a circle of radius unity and centre $\frac{-1}{2}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive C.R equations in polar coordinates.

Or

(b) If $u(x, y) = ax^2 - y^2 + xy$ is harmonic, find the value of 'a'. Find an analytic function $f(z)$ for which u is the real part.

17. (a) State and prove Cauchy-Goursat theorem.

Or

(b) Evaluate :

(i) $\int_C \frac{\sin z \, dz}{\left(z - \frac{\pi}{2}\right)^2}$, where C is the circle $|z| = 2$.

(ii) $\int_C \frac{z^3 dz}{(2z + i)^3}$ where C is the unit circle.

18. (a) State and prove Laurent's series.

Or

(b) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series

(i) about the point $z = 0$ and

(ii) about the point $z = 1$. Determine the region of convergence in each case.

19. (a) Evaluate : $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$.

Or

(b) Using the method of Contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)\,(x^2+4)} dx$.

20. (a) Prove that the set of all bilinear transformations is a group under composition.

Or

(b) Prove that the cross ratio of four points is real when the points lie on a circle.